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A discrete λ-medial axis

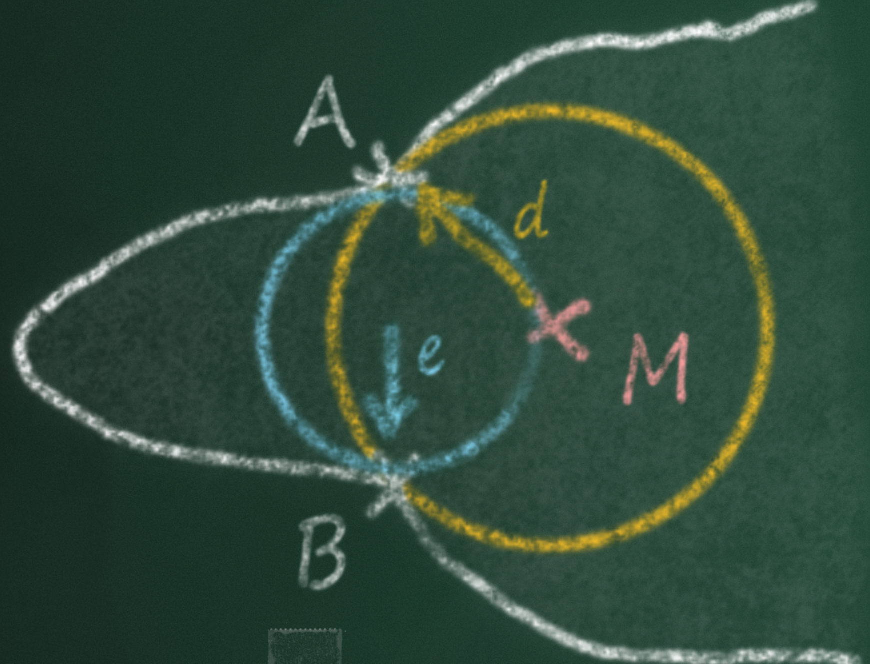
I. Definition

The Discrete Lambda Medial Axis (DLMA) is the discrete version of the Lambda Medial Axis initially proposed by Chazal and Lieutier. The goal is to obtain filtered skeletons which are stable with respect to small perturbations of object's contour.

Given $X \in \mathbb{Z}^n$ and $m \in X$, the extended projection of m on $\overline{X}$, denoted by $E_{\overline{X}}(m)$, is the set of points of $\overline{X}$ which are the closest to m or to one of its neighbours.
We denote by L_X the map of X to $\mathbb{R}$ such that, for all $m \in X$,

$L_X(m) = \mathrm{Min}\{ r \in \mathbb{R} \,/\, \text{there is a ball } B \text{ of radius } r \text{ such that } E_{\overline{X}}(m) \subseteq B \}$.

L_X is called the DLMA map of X. Given $\lambda \in \mathbb{R}$, the discrete λ-medial axis of X, denoted by $DLMA_X(\lambda)$, is the set of points m of X such that $L_X(m) \geq \lambda$.



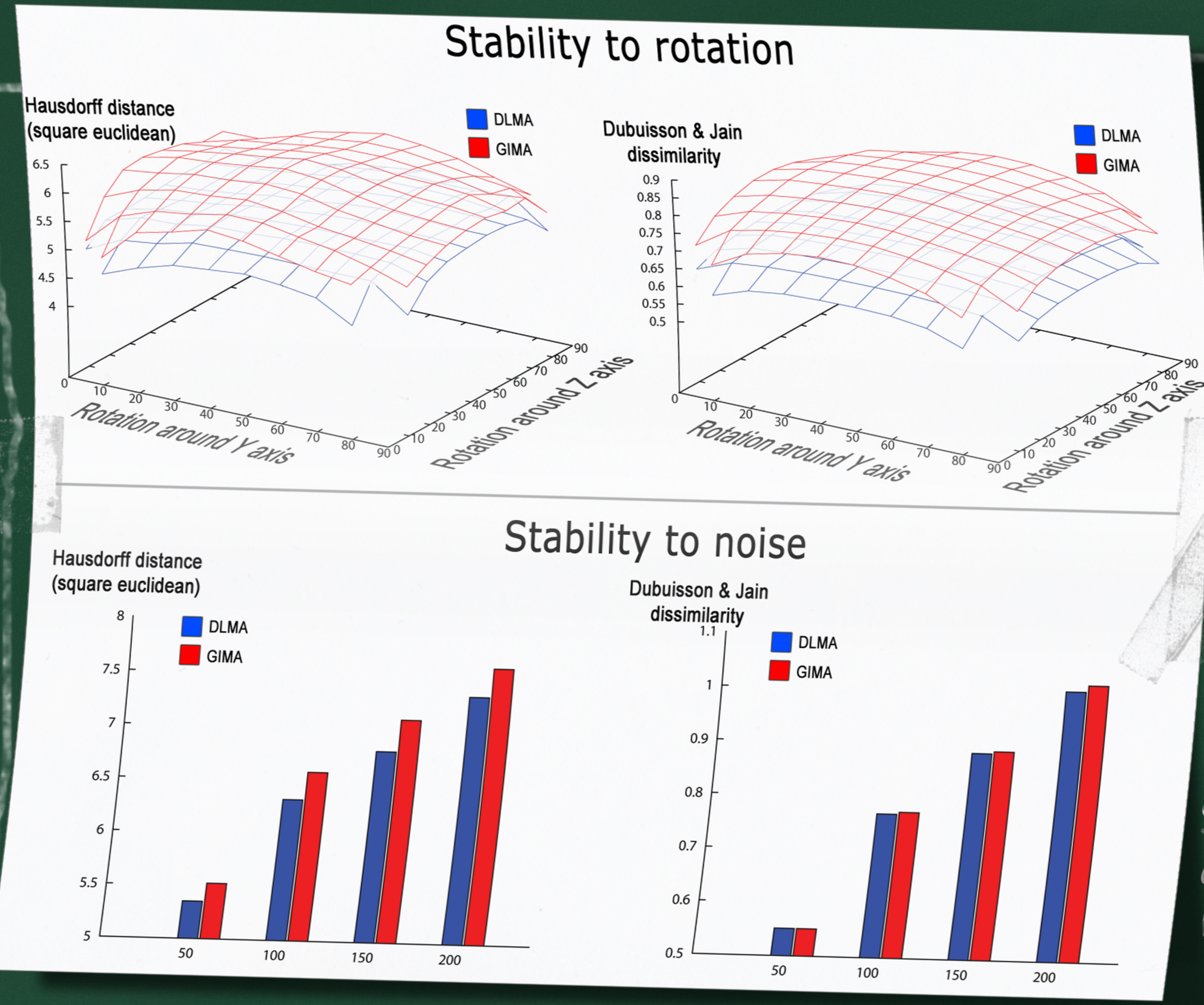
On the figure on the left, we have $E_{\overline{X}}(M) = \{A,B\}$ – both points are at distance d from M.
The smallest disk containing A and B is the blue disk, of radius e : $L_X(M) = e$.

Examples of DLMA map

Original objects (2d and 3d)

DLMA maps (red is high value, blue is low value)

II. Comparing with other algorithm



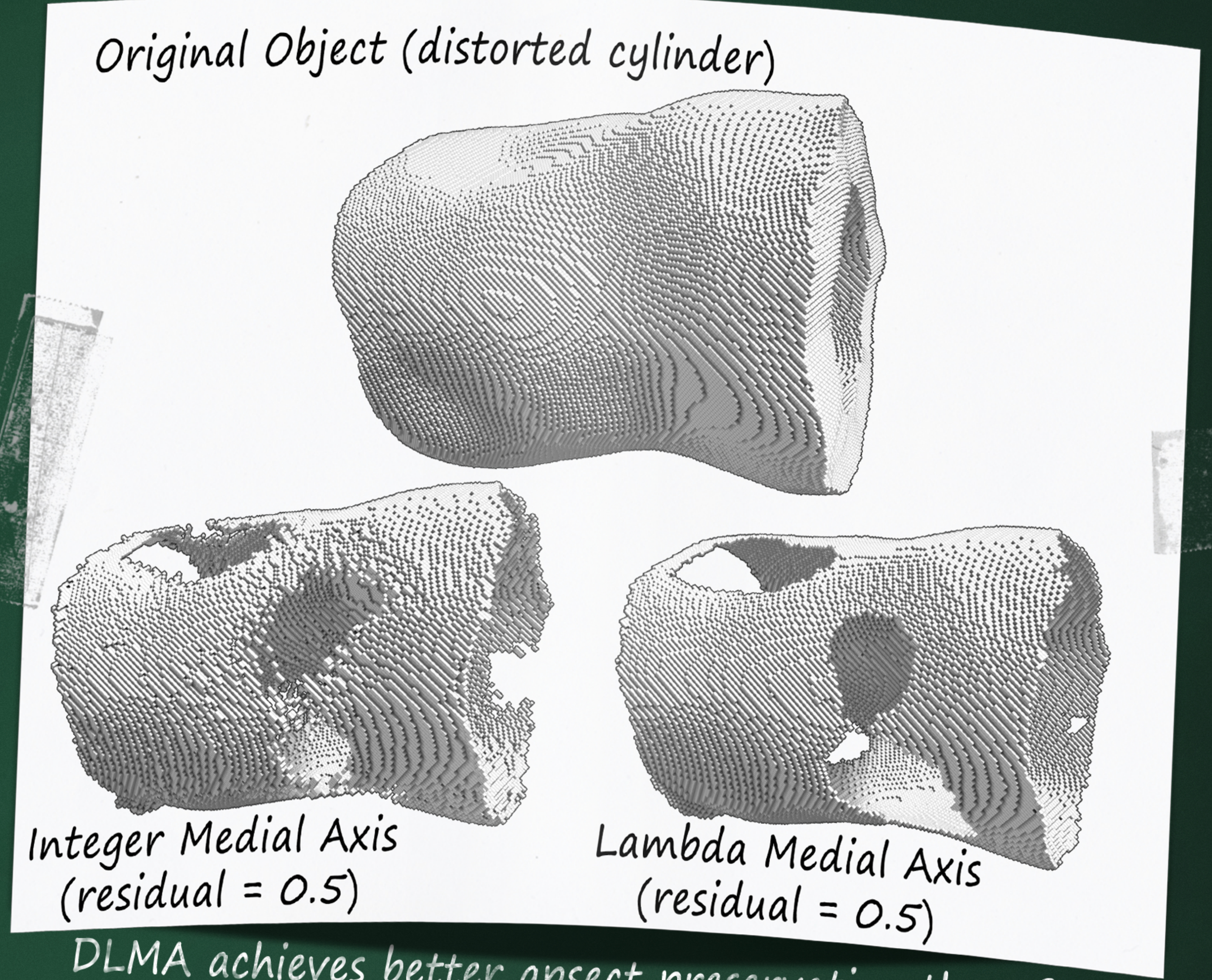
Stability to noise and rotation, for a residual of 5%.

We compare our DLMA with another "medial axis method" called the integer medial axis (GIMA), introduced by Hesselink and Roerdink. The GIMA algorithm takes as input a set X and a filtering parameter γ and produces a subset of X.

We can only compare both methods when they produce comparable results. The residual of the reconstruction based on a subset Y of X, represents the quantity of information missed when trying to rebuild X from Y. We will say that $DLMA_X(\lambda)$ and $GIMA_X(\gamma)$ are comparable if the reconstructions based on these sets produce the same residual.

We compare the stability of both algorithms to rotation. Given a set X, we compute the parameters λ and γ so that GIMA and DLMA can be compared. Let R be a discrete rotation transformation, we compute the dissimilarity between $DLMA_{R(X)}(\lambda)$ and $R(DLMA_X(\lambda))$, and between $GIMA_{R(X)}(\gamma)$ and $R(GIMA_X(\gamma))$. The same process is done with noise addition.

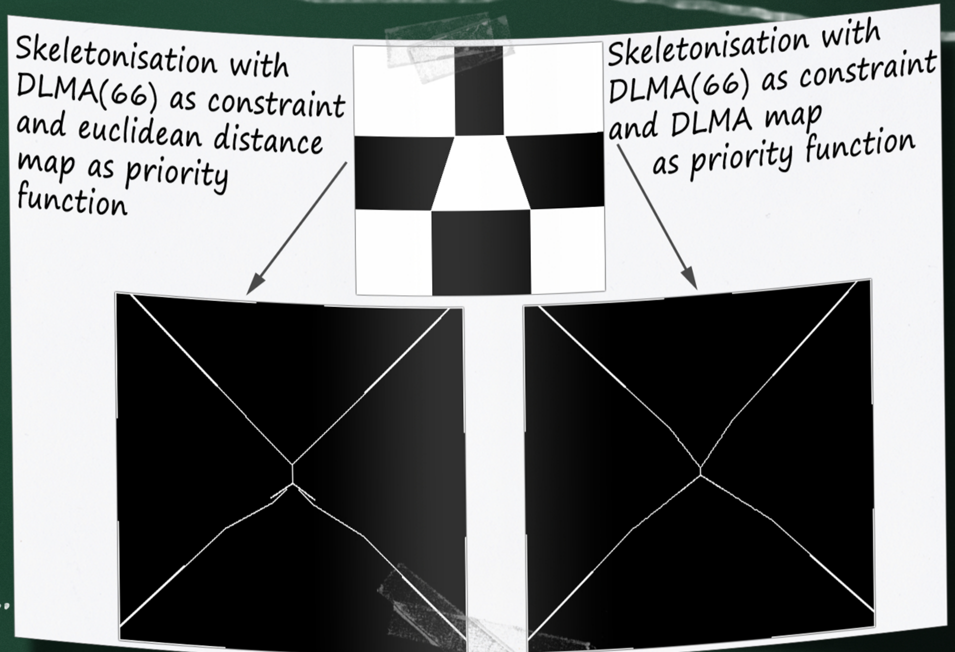
Results (above) show that DLMA is more robust than GIMA to noise and rotation. On the right, we show different results obtained with the GIMA and the DLMA (filtered at convenient values) methods. Results show that, when the object has irregular thickness, the DLMA achieves better visual aspect preservation.



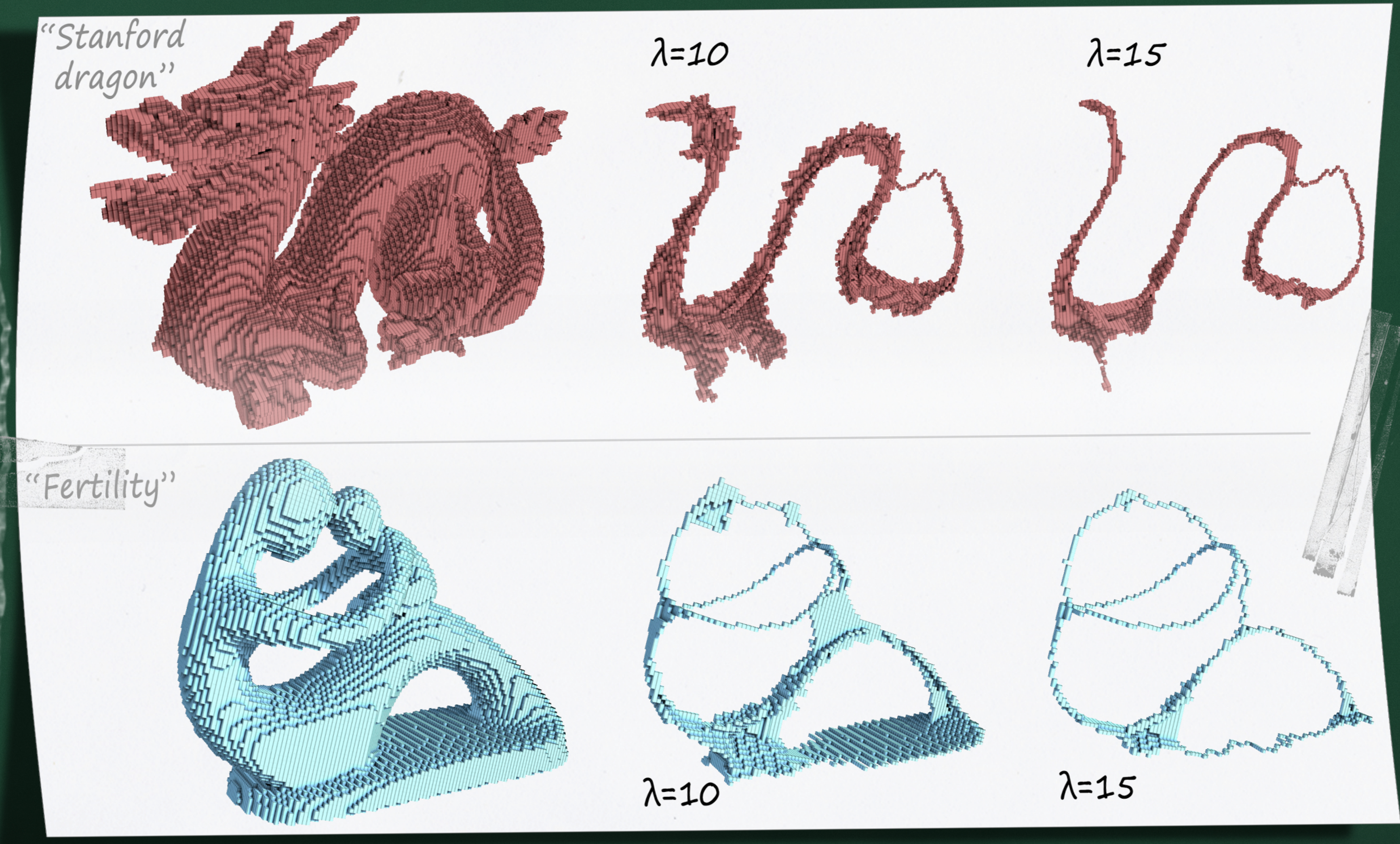
DLMA achieves better apsect preservation than GIMA

III. Homotopic thinning and results

DLMA and GIMA do not preserve homotopy of the input. An homotopic skeleton is obtained by removing iteratively simple points from input object, until no more simple point can be found.



Unwanted pattern appears in the centre when using distance map as priority function.



Some skeletons obtained using DLMA(10) and DLMA(15) as constraints.

The filtered DLMA can be used as a constraint set during skeletonisation: no point of the DLMA will be removed during the thinning. When using the DLMA (filtered at a convenient value) as a constraint, one should also use the DLMA map as priority function for points removal (see figure above, where DLMA and distance map were used as constraints: better results are obtained when using the DLMA map as priority function – look at the center of the results).

On the left, we show skeletons obtained on various objects, all using a DLMA as constraint (filtered at values 10 and 15).